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Holographic entanglement entropy of confining gauge theories with flavor

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ABSTRACT

We compute the holographic entanglement entropy for confining gauge theories with matter fields using the formula of Ryu and Takayanagi. The gravity solutions of our interest are the wrapped D5-brane solutions of Maldacena and Nunez, and the generalizations with extra matter fields. We obtain the relation between the entanglement entropy vs. size of the subsystem, and find that the critical length is increased as we add more matter fields.

1. Introduction

Entanglement entropy (EE) has turned out to be a very useful quantity in many physical systems. In order to define EE, one first needs to divide the space into two subspaces. The boundary surface between two subsystems Σ is called the entangling surface. EE of a quantum system is then defined as the von Neumann entropy of the reduced density matrix which is obtained by tracing over the degrees of freedom outside the subsystem we are interested in.

EE is dominated by short-distance physics across Σ, and if we call the UV cutoff ϵ, the leading divergent term for $(d+1)$-dimensional quantum field theory is proportional to the area of Σ divided by ϵ^{d-1} [1]. It is a rule of thumb that the coefficient of this divergent term depends on the regularization method employed and does not contain universal data. On the other hand, the subleading terms in EE can provide cutoff independent and important information about the field theory. For conformal field theories in even dimensions there are logarithmic divergent terms $\log(\epsilon)$ and the coefficient is related in a definite way to the central charge, thus the physical degrees of freedom [2]. As a simple and concrete example, when the entangling surface Σ is simply a flat hyperplane separated by l, a massless scalar field theory gives [3]

$$S = \frac{n}{24\pi}\left(\frac{L^2}{\epsilon^2} - \frac{L^2}{l^2}\right), \tag{1}$$

where n is the number of real scalar fields. But for interacting quantum field theory and generic entangling surface Σ, the first-principle computation of EE is hard. We refer the readers to [4] and references therein, for EE in free field theories.

For strongly coupled quantum field theories which have gravity dual, a holographic prescription for EE was proposed in [5]. For $(d+1)$-dimensional boundary theory and a spatial region V whose boundary is the entangling surface Σ, EE is calculated by minimizing the bulk surface area

$$S = \frac{\mathrm{Area}(\gamma_\Sigma)}{4G_N^{(d+2)}}, \tag{2}$$

where γ_Σ is a d-dimensional static minimal surface with boundary Σ. γ_Σ is extended into bulk geometry which is AdS_{d+2} for conformal field theories.

For string theory backgrounds we usually have internal space as well as anti-de Sitter part, over which one needs to integrate over as well. The general formula is then

$$S_A = \mathrm{Min}\,\frac{1}{4G_N^{(10)}}\int_{\gamma_A} d^8x\,\sqrt{-g}e^{-2\phi}. \tag{3}$$

And the integral should be minimized for the surfaces whose boundary coincides with the boundary of the region we want to calculate the EE. Note that here the metric g is induced metric on surface γ, from *string frame* metric. ϕ is of course the dilaton. The Newton constant is related to string length as follows

$$G_N^{(10)} = 8\pi^6\alpha'^4. \tag{4}$$

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Then an explicit calculation for e.g. $\mathcal{N}=4$ super Yang–Mills theory gives exactly the same behavior as (1), although there is a mismatch of numerical coefficient by about 3/2 [3].

When the holographic prescription of EE produces results in harmony with solvable or maximally supersymmetric examples [5], it is natural to try to extend the comparison to more nontrivial examples of gauge/gravity correspondence. In this Letter we are interested in the gauge/gravity examples away from the conformal fixed point. In addition to the consideration of string theory p-branes in [3], confining gauge theory duals such as compactified D4-branes and Klebanov–Strassler backgrounds were considered in [6]. Gauge/gravity pairs exhibiting nonlocality such as little string theory and noncommutative field theory were studied in [7]. See also [8,9] for more recent works on nonconformal examples.

In this Letter we choose a well-known example of confining gauge/gravity example, the Maldacena–Nunez (MN) solution of IIB supergravity [10]. This solution is obtained by considering D5-branes wrapped on S^2 in Calabi–Yau three-fold, and unlike the flat D5-brane solution, it is completely regular. Since S^2 is topologically trivial the $3+1$-dimensional field theory is pure $\mathcal{N}=1$ supersymmetric QCD. The reason why we choose especially this solution in this work is because there exists a rather thorough study of how to extend this theory with matter fields, see [11–16]. As we will see below, the behavior of EE as a function of the size of the entanglement subsystem for MN is similar to the results presented in [6]. The regularized EE is negative but increases as the size is increased up to some critical distance ℓ_{crit}, like (1) after the UV divergence is removed. For $\ell > \ell_{crit}$ EE is constant and zero, thus exhibiting *screening* behavior. The main point of this Letter is to consider flavor-extended MN models and see how ℓ_{crit} changes as we increase the number of matter fields. According to our analysis, ℓ_{crit} gets bigger when we have more matter fields, just as it should be since matter fields make the theory less confining.

This Letter is organized as follows. In Section 2 we study EE of Maldacena–Nunez background and compare it to the result of Klebanov–Strassler backgrounds [17]. In Section 3 we consider the generalization to flavored MN models. We conclude in Section 4.

2. EE in Maldacena–Nunez $\mathcal{N}=1$ SQCD dual

Let us start with the nontrivial gravity background dual to confining gauge theory suggested by Maldacena and Nunez (MN) [18]. Physically what is being considered is a twisted compactification of D5-branes when they are wrapped on a supersymmetric cycle S^2 in Calabi–Yau manifold. Due to the trivial topology of S^2, it is expected that at low energy the worldvolume theory is simply supersymmetric QCD without any matter fields. Mathematically the MN construction is built on the gravitating BPS monopole solutions in [19] and it is a regular solution, unlike the near-horizon limit of NS5-branes or D5-branes.

The metric we will use, for wrapped D5-branes, is given as

$$ds^2_{str} = e^{\phi_D}\left[dx_4^2 + N\alpha'\left(d\rho^2 + e^{2g(\rho)}\,d\Omega_2^2 + \frac{1}{4}\left(\omega^a - A^a\right)^2\right)\right], \tag{5}$$

with

$$e^{2\phi_D} = e^{-2\phi_0}\frac{\sinh 2\rho}{2e^{g(\rho)}}, \tag{6}$$

$$e^{2g(\rho)} = \rho\coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4}. \tag{7}$$

We note that ω^a, $a=1,2,3$, are left-invariant 1-forms for S^3 and A^a are gauge 1-forms on three-dimensional space parametrized by ρ and the two-sphere $d\Omega_2 = d\theta^2 + \sin^2\theta\, d\phi^2$. They are given as

$$A^1 = \frac{1}{2}a(\rho)\,d\theta, \qquad A^2 = a(\rho)\sin\theta\,d\phi, \qquad A^3 = \cos\theta\,d\phi \tag{8}$$

with $a(\rho) = 2\rho/\sinh 2\rho$, but the concrete form of A^a does not affect the calculation of EE.

Now it is straightforward to write down the action for extremal surface with the shape of a strip with width ℓ, starting from the definition (3).

$$S = \frac{N^3L^2e^{-2\phi_0}}{8\pi^3\alpha'}\int_{-\ell/2}^{\ell/2} dx\, e^{g(\rho)}\sinh 2\rho\sqrt{\dot\rho^2 + \frac{1}{N\alpha'}}. \tag{9}$$

Following the notation of [6], it is useful to define $H(\rho) = e^{g(\rho)}\sinh 2\rho$ which governs the extremization problem at hand. This function is monotonically increasing in the range of $0 < \rho < \infty$.

Using the "energy" integral, we find that the width ℓ and the minimum value ρ_* are related as follows.

$$\frac{\ell}{2} = \sqrt{N\alpha'}\int_{\rho_*}^{\infty}\frac{H(\rho_*)\,d\rho}{\sqrt{H(\rho)^2 - H(\rho_*)^2}}. \tag{10}$$

We have evaluated this integral numerically and a plot is given in Fig. 1. One can see that $\ell(\rho_*)$ is monotonically increasing, and the limiting behavior is

$$\ell(\rho) = \sqrt{N\alpha'}\left(\frac{\pi}{2} - \frac{0.365}{\rho_*} + \cdots\right), \qquad \rho_* \to \infty. \tag{11}$$

The limit value $\ell_{max} = \sqrt{N\alpha'}\pi/2$ is the Hagedorn length scale which appeared before in the study of confining gauge duals [17]. The MN background can be also thought of as a deformation of NS5-brane theory. It was pointed out earlier [3,7] that the consideration of holographic EE for NS5-branes has a rather peculiar behavior, in the sense that the smooth minimal surface exists only at a particular value of slab width ℓ, and our result is exactly the same.

When we compare Fig. 1 against analogous graphs in [17], we find they are qualitatively different. For the models studied in [17], $\ell(\rho_*)$ is not monotopic: it starts from $\ell(0)=0$, increasing at first as we increase ρ_*, but after hitting the maximum value at a finite value of ρ_* it decreases to zero as $\rho_* \to \infty$. This implies that the inverse function $\rho_*(\ell)$ is double-valued, so in general there are two locally minimizing surfaces for $\ell < \ell_{max}$.

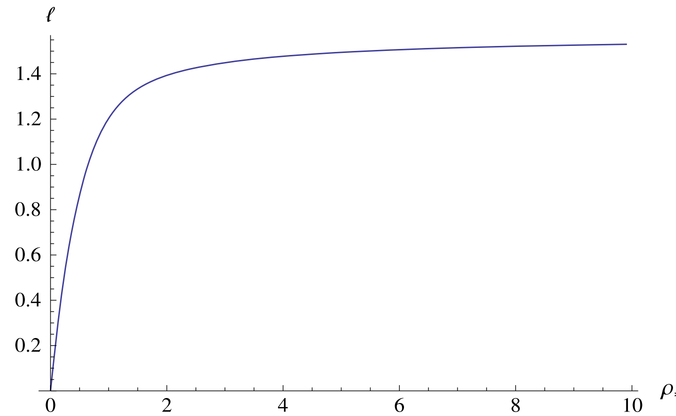

Fig. 1. The plot of strip width vs. minimum value of ρ for volume-minimizing surface, for MN solution.

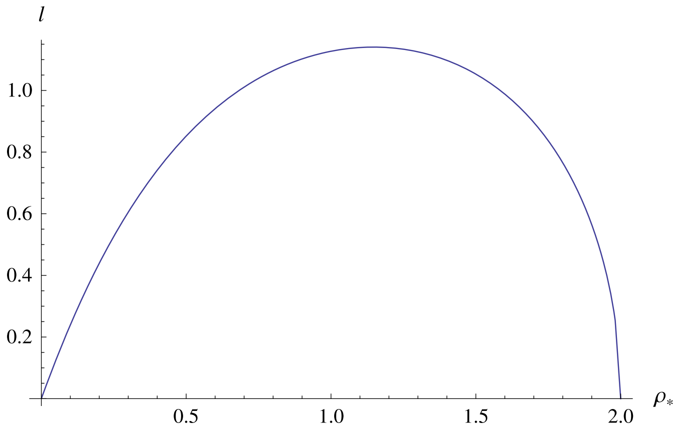

Fig. 2. The plot of strip width vs. minimum value of ρ for MN solution with explicit cutoff at $\rho_{max} = 2$.

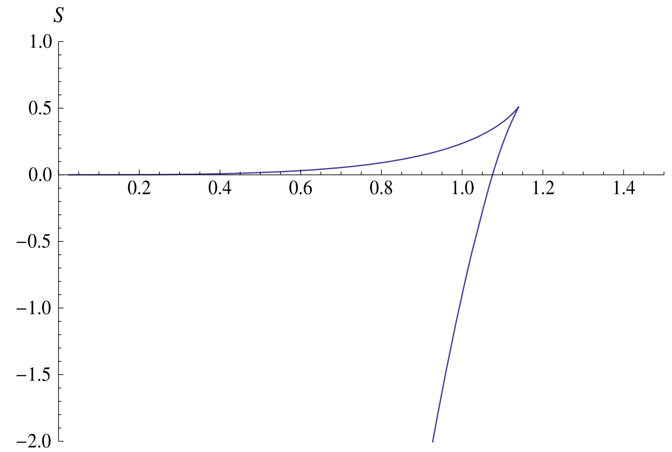

Fig. 3. Holographic entanglement entropy vs. width of slab.

It turns out that the globally minimal surface is from the branch of *larger* ρ_*, for all the examples in [17]. But this is the physically relevant part which is missing in Fig. 1. We interpret this as the breakdown of MN solution for large ρ. In fact, for large ρ the dilaton field of MN wrapped D5-brane solution is divergent, so it is suggested one should replace it with the S-dual description of NS5-branes [10]. When it comes to the supergravity solutions D5-branes and NS5-branes give rise to the same solution in Einstein frame. We posit that the reason we obtain unphysical $\ell(\rho_*)$ relation is that the string modes are not decoupled from supergravity modes, as discussed in [10]. So instead of using the MN solution all the way up to $\rho_* \to \infty$, we introduce a hard-wall type UV cutoff ρ_m. This prescription is rather reminiscent of the simple-minded AdS/QCD prescription of the same name, see for instance [20]. It is applied to the holographic EE of NS5-brane theory in [7]. As we will see below, this simple prescription gives reasonable results for holographic EE.

With cutoff ρ_m the range of ρ is limited to $0 < \rho < \rho_m$, and

$$\frac{\ell}{2} = \sqrt{N\alpha'} \int_{\rho_*}^{\rho_m} \frac{H(\rho_*)\, d\rho}{\sqrt{H(\rho)^2 - H(\rho_*)^2}}. \tag{12}$$

For ρ_m we choose a value where e^{ϕ_D} is sufficiently large. For concreteness we set $\rho_m = 2$ where $\sinh(2\rho)e^{-g(\rho)} \approx 400$. $\ell(\rho_*)$ is plotted in Fig. 2.

We now substitute the energy integral into the EE formula and "regularize" the expression by subtracting the area of disconnected surfaces. The quantity we are interested in is

$$S_{reg} = \frac{N^3 L^2 e^{-2\phi_0}}{4\pi^3 \alpha'} \left[\int_{\rho_*}^{\rho_m} \frac{H(\rho)^2}{\sqrt{H(\rho)^2 - H(\rho_*)^2}}\, d\rho - \int_0^{\rho_m} H(\rho)\, d\rho \right]. \tag{13}$$

This quantity starts from zero at $\rho_* = 0$, and negative at $\rho_* = \rho_m$. It is then obvious that large ρ_* branch gives the globally area-minimizing surfaces. The numerical plot is given in Fig. 3. One can see that for the lower curve the regularized EE is negative for $\ell < \ell_{crit} \approx 1.07$. For $\ell > \ell_{crit}$ the smooth regular surface either does not exist or it has larger volume compared to flat hyperplane in the bulk. The EE at zero width $S_{reg}(\ell = 0) \approx -23$ in this case is negative and finite. This is certainly different from the previous computations of AdS_5 [3] or confining backgrounds in [6], but it is understandable since we here introduced an explicit UV cutoff ρ_m.

3. EE in flavored Maldacena–Nunez solutions

3.1. Flavored MN solutions

As it is well known, the original AdS/CFT conjecture involves maximally supersymmetric examples [21] and not very attractive in the phenomenological sense. Adding branes is a convenient way of considering gravity dual of gauge theories with matter fields, and there is a huge literature devoted to this subject. An earlier work can be found in [22]. The extension of MN solution with fundamental representation matter, preserving the same supersymmetry, is studied by C. Nunez and collaborators in a series of papers [11–16].

The fully backreacted solutions which account for the addition of matter fields are given as follows [14]

$$\begin{aligned} ds^2 = \alpha' e^{\phi(\rho)/2} \bigg\{ & \frac{1}{\alpha'} dx_{1,3}^2 + Y(\rho)\big(4d\rho^2 + (\omega_3 + \tilde\omega_3)^2\big) \\ & + \frac{P(\rho)}{2\sinh(2\rho)} (\omega_1 \tilde\omega_1 - \omega_2 \tilde\omega_2) \\ & + \frac{1}{4}\big(P(\rho)\coth(2\rho) + Q(\rho)\big)\big(\omega_1^2 + \omega_2^2\big) \\ & + \frac{1}{4}\big(P(\rho)\coth(2\rho) - Q(\rho)\big)\big(\tilde\omega_1^2 + \tilde\omega_2^2\big) \bigg\}. \end{aligned} \tag{14}$$

The undetermined functions are still dependent only on the radial coordinate ρ. Here $\tilde\omega^a$ are left-invariant 1-forms for $SU(2)$ satisfying $d\tilde\omega^a = \frac{1}{2}\epsilon^{abc}\tilde\omega^b \wedge \tilde\omega^c$, and $\omega^1 = d\theta$, $\omega^2 = \sin\theta\, d\phi$, $\omega^3 = \cos\theta\, d\phi$. The solution is purely D5-brane configuration, in the sense it only involves metric, dilaton and RR 3-form field.

It is shown in [14] that the condition for supersymmetry for the above ansatz can be reduced to a single *master equation*:

$$\begin{aligned} P'' + \big(P' + N_f\big)\bigg(& \frac{P' + Q' - 2N_f}{P - Q} + \frac{P' - Q' + 2N_f}{P + Q} \\ & - 4\coth(2\rho)\bigg) = 0. \end{aligned} \tag{15}$$

Here Q is pre-determined as (for $N_f < 2N_c$)

$$Q = \frac{2N_c - N_f}{2}\big(2\rho\coth(2\rho) - 1\big). \tag{16}$$

Then the rest of the solution is determined in terms of P, Q as follows.

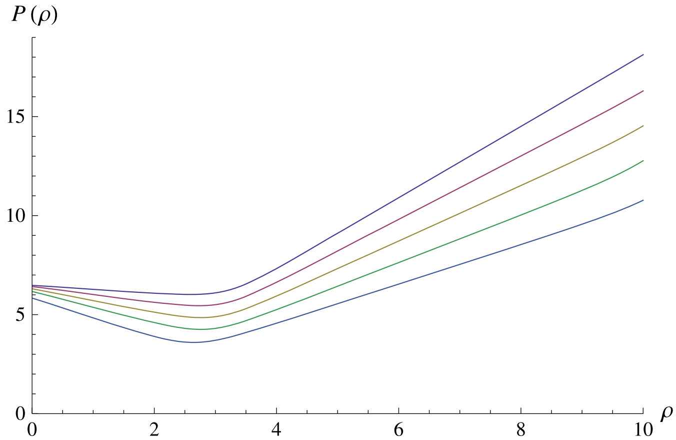

Fig. 4. Numerical solutions for P, with $N_f/N_c = 0.2, 0.4, 0.6, 0.8, 1.0$ from top to bottom.

$$Y = \frac{1}{8}(P' + N_f),$$
$$e^{2\phi} = \frac{\sinh 2\rho}{\sqrt{(P^2 - Q^2)Y}}. \tag{17}$$

The flux fields are also determined once P is fixed but we do not present them here since the calculation of EE does not need that information.

Here we are interested in the so-called type **N** backgrounds, in the classification criterion of [14]. The master equation is non-linear differential equation and does not allow series expansion method, but when we ignore $\mathcal{O}(e^{-4\rho})$ terms it can be checked [14] that there should exist solutions which behave for large ρ as

$$P = Q + N_c\left(1 + \frac{N_f}{4Q} + \frac{N_f(N_f - 2N_c)}{8Q^2} + \mathcal{O}(Q^{-3})\right), \tag{18}$$

$$Y = \frac{N_c}{4} - \frac{N_c N_f}{(64N_c - 32N_f)\rho^2} + \mathcal{O}(\rho^{-3}), \tag{19}$$

$$e^{4\phi} = e^{4\rho}\left(\frac{1}{(4N_c^3 - 2N_c^2 N_f)\rho} + \frac{2N_c - 3N_f}{8N_c^2(-2N_c + N_f)^2\rho^2} + \mathcal{O}(\rho^{-3})\right). \tag{20}$$

It is argued in [14] that such solutions provide nice description of non-perturbative physics of SQCD with flavor, for instance it is shown that for such solutions the gaugino condensate is non-zero in dual field theory.

The numerical solutions for P with asymptotic behavior given as (18) were presented in [15], see Fig. 7 in that paper. Here we have reproduced the numerical solutions for several values of $0 < N_f/N_c < 1$, using (18) to set the initial condition for some large value of ρ. Our subsequent analysis will be based on the numerical solutions in Fig. 4.

3.2. Calculation of EE

Now it is straightforward to consider the volume of EE surface

$$S = \frac{L^2}{8\pi^3\alpha'}\int_{-\ell/2}^{\ell/2} dx\, e^{2\phi}(P^2 - Q^2)\sqrt{Y(4Y\dot{\rho}^2 + 1/\alpha')}. \tag{21}$$

The original Maldacena–Nunez background corresponds to $P = 2N_c\rho$ and it is easy to check that (21) reduces to (9).

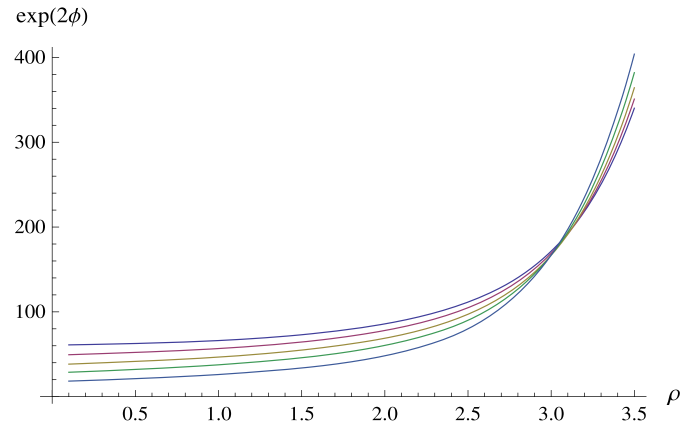

Fig. 5. Behavior of dilaton for different N_f/N_c.

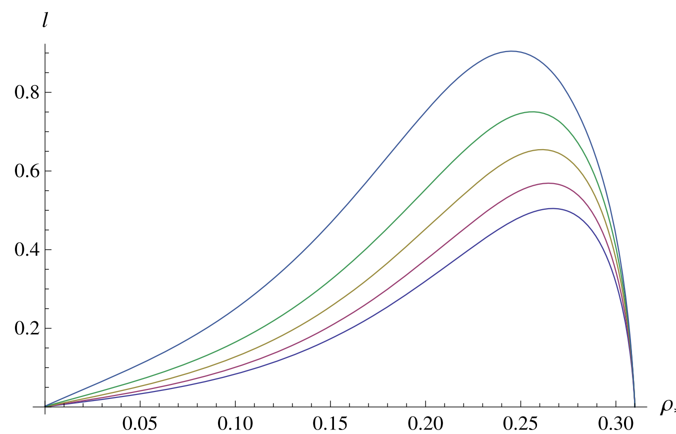

Fig. 6. Width of slab vs. ρ_*. ℓ_{max} is larger for large values of N_f/N_c.

For convenience we define

$$H(\rho) = e^{2\phi}Y(P^2 - Q^2), \qquad \beta(\rho) = \frac{1}{4Y}. \tag{22}$$

And we may proceed in the same way as before. The slab width is

$$\frac{\ell}{2} = \int_{\rho_*}^{\rho_m} d\rho\, \frac{H_*\sqrt{\beta_*}}{\sqrt{H^2\beta^2 - H_*^2\beta_*\beta}} \tag{23}$$

where we abbreviated $H = H(\rho)$, $H_* = H(\rho_*)$, $\beta = \beta(\rho)$, $\beta_* = \beta(\rho_*)$. And EE is

$$S_{reg} = \frac{N^3L^2}{4\pi^3\alpha'}\left[\int_{\rho_*}^{\rho_m} \frac{H^2\beta}{\sqrt{H^2\beta^2 - H_*^2\beta_*\beta}}\, d\rho - \int_0^{\rho_m} H\, d\rho\right]. \tag{24}$$

Now we need to choose the UV cutoff ρ_m. In general its value does not have to be the same for different N_f/N_c, since the dilaton flows differently. As a reasonable solution to this ambiguity, we plotted $e^{2\phi}$ and checked if there is a suitable choice for ρ_m. From Fig. 5 we see that at $\rho_m \approx 3.1$ dilaton has almost the same value independent of N_f/N_c. We have performed the integration numerically and the result is presented in Fig. 6 for $\ell(\rho_*)$. It shows that for large N_f/N_c, the critical width ℓ_{crit} becomes larger. This is a reasonable result, since adding more matter fields to the system renders the one-loop beta function less negative. That will decrease the QCD scale Λ_{QCD}, so as inverse of that energy scale ℓ_{crit} will increase. We also present $S(\ell)$ in Fig. 7.

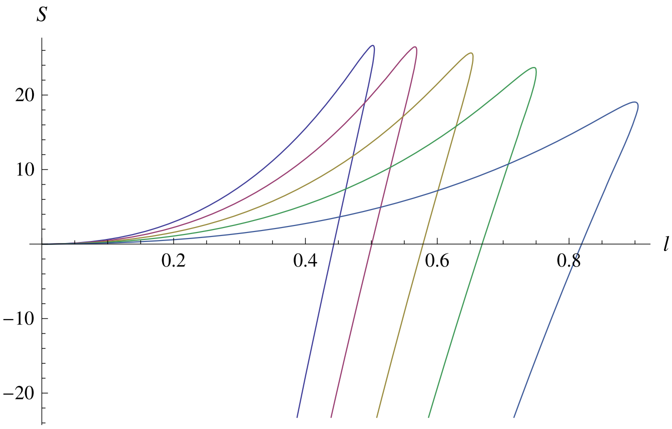

Fig. 7. EE vs. ℓ. ℓ_{crit} is larger for large values of N_f/N_c.

4. Discussion

In this Letter we have considered holographic EE of confining gauge theory duals. In particular, we have chosen the Maldacena–Nunez background and its deformation with extra flavor fields. The main motivation of this work was to see the trend of EE's behavior as we change the physically important parameter N_f/N_c. We have obtained a reasonable result that EE behaves similarly to weakly coupled field theory at short distance, and ℓ_{crit} signifying the transition into screening phase increases as we add more matter fields.

Recently there was a paper which also studied EE and how it changed as a function of parameters defining the confining gauge theory [9]. Our analysis can be viewed as complementary to it. It will be interesting to consider adding more parameters into the flavored MN model and calculate EE. For instance one can consider putting MN background at finite temperature, and study EE. There exists for instance a finite temperature study of MN in [23], for the special case of $N_f = 2N_c$. It will be certainly viable to generalize this study to generic values of N_f/N_c, and perform holographic analysis like computation of EE.

It will be also interesting to study EE of flavored Klebanov–Strassler model. The addition of matter fields is considered in [24–26]. We plan to address this problem in a separate publication.

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